

REVIEW PROBLEMS FOR INTERMEDIATE ALGEBRA ASSESSMENT TEST-Rev 1

1. Simplify and combine like terms: $-18x + 3(2x - 8) - 9x - (4 - 6x)$
2. Multiply: $(2z^3 - 4z^2 + 6z - 1)(3z + 2)$
3. Multiply: $(2a + 3)(4a^2 - 6a + 9)$
4. Multiply: $(5x - 2)(3x + 4)$
5. $3\sqrt{25} =$
6. $\frac{6^2 y^3 z^5}{6^4 y^2 z^7} =$
7. $\frac{5^0 (x^2)^3 y}{5x(y^2)^5} =$
8. $\frac{7a^3(b + c)^2}{14a(b + c)} =$
9. $3^{2^4} =$
10. $16^{\frac{5}{4}} =$
11. Simplify. All exponents must be positive.
$$\left(\frac{x^{-\frac{3}{5}} z^{-1}}{2^{-2} z^{-\frac{1}{2}} x} \right)^{-1} =$$
12. Solve: $7z - 3z + 12 - 5 = 8z + 7$
13. Solve: $-4(3 + 2m) - m = -3m$
14. Solve for x: $Q = 3x + 2y + 4z$
15. Solve for x: $4^x = 8^{x-3}$
16. Scott Hardy invested some money at 12% and \$4000 less than this amount at 14%. Find the amount invested at each rate if his total annual interest income is \$4,120.
17. A pharmacist needs 100 liters of a 50% alcohol solution. She has a 30% alcohol solution and an 80% alcohol solution that she can mix. How many liters of each does she need?
18. Two people need to sort a pile of bottles at the recycling center. Working alone, one person could do the entire job in 9 hours, and the other person could do the entire job in 6 hours. How long would it take them to complete the job if they work together?

19. Solve the system: $2x + 3y = 10$
 $-3x + 2y = 11$
20. Solve the system: $3x + 4y = 8$
 $6x = 7 - 8y$
21. Steve and Ross must paint Steve's entire house. Steve bought 3 gallons of paint and 2 brushes at one store for \$48. Ross bought 5 gallons of paint and 1 brush at another store for \$66. Assuming the price per gallon and cost per brush were the same at both stores, find what they paid per gallon for paint and the cost per brush.
22. Solve: $\frac{2q + 1}{3} - \frac{q - 1}{4} = -2$
23. Solve: $5x^2 - 3x = 0$
24. Find the roots of the following equation by factoring: $10x^2 - 13x = 3$
25. Find the roots of the following equation using the quadratic formula:
 $2x^2 + 3x = 10$
26. Find the roots of the following equation: $(2x + 1)(x - 2) = -3$
27. Solve: $\frac{y}{3} - \frac{y}{7} = 1$
28. Solve: $\frac{1}{m + 4} - \frac{3}{2m + 8} = \frac{1}{2}$
29. Rationalize the denominator and simplify: $\frac{-6\sqrt{3}}{\sqrt{2}}$
30. Multiply and combine like terms: $(2x + 3)^2$
31. Multiply and combine like terms: $(2x + 3)^2 - (x + 2)^2$
32. Evaluate the following expression if $x = 3$ and $y = -2$.
 $3x^2y - 4xy^2$
33. Find the distance between the point $(-4, -2)$ and the point $(6, -1)$.

34. Divide: $\frac{8m^4 - 6m^3 + 2m}{2m}$

35. Divide: $(15p^2 + 11p - 17) \div (3p - 2)$

36. What quadrant is the point $(-3, 10)$ in?

37. Reduce, if possible: $\frac{6x^2 + 7x - 5}{3x + 5}$

38. Multiply: $\frac{2p^2 - 5p - 12}{5p^2 - 18p - 8} \cdot \frac{25p^2 - 4}{30p - 12}$

39. Divide: $\frac{25p^3q^2}{8p^4q} \div \frac{15pq^2}{16p^5}$

40. Divide: $(y^2 + 2y) \div \frac{y + 5}{y^2 + 4y - 5}$

41. Combine: $\frac{3}{t - 2} - \frac{5}{2 - t}$

42. Combine: $\frac{2}{y + 1} + \frac{6}{y - 1}$

43. Combine: $\frac{3r}{10r^2 - 3rs - s^2} + \frac{2r}{2r^2 + rs - s^2}$

44. Simplify: $\frac{\frac{3}{x} - 5}{6 + \frac{1}{x}}$

45. Sketch the graph of the line $3x - y = 4$.

46. If the slope, m , of line is $\frac{1}{2}$, and the y - intercept is at $(0, 3)$, what is the equation of the line?
Express the answer in slope-intercept form.
47. What is the slope of the line between the two points $(4, -3)$ and $(-2, -1)$?
48. What is the equation of the line with slope $= \frac{2}{3}$ and containing point $(2, -1)$?
Express the answer in standard form.
49. What is the equation of the line which goes through the two points $(3, 5)$ and $(-1, 0)$? Express the answer in standard form.
50. Sketch the graph of the line $y = -2$.
51. Solve $\sqrt{x+2} = x - 4$
52. Solve $|x - 2| < 3$
53. $4! =$
54. Solve $-3x > 12 + x$

SOLUTIONS FOR INTERMEDIATE ALGEBRA ASSESSMENT TEST

1. $-18x + 3(2x - 8) - 9x - (4 - 6x)$

Use the distributive law to remove parentheses:

$$-18x + 6x - 24 - 9x - 4 + 6x$$

Combine like terms: $-15x - 28$

2. Multiply vertically to easily line up like terms:

$$\begin{array}{r} 2z^3 - 4z^2 + 6z - 1 \\ \underline{3z + 2} \\ 6z^4 - 12z^3 + 18z^2 - 3z \\ \underline{4z^3 - 8z^2 + 12z - 2} \\ 6z^4 - 8z^3 + 10z^2 + 9z - 2 \end{array}$$

3. Multiply vertically as in #2.

$$\begin{array}{r} 4a^2 - 6a + 9 \\ \underline{2a + 3} \\ 8a^3 - 12a^2 + 18a \\ \underline{+ 12a^2 - 18a + 27} \\ 8a^3 \qquad \qquad + 27 \end{array}$$

answer: $8a^3 + 27$

4. Multiply using the acronym FOIL (firsts, outsides, insides, lasts)

$$\begin{aligned} (5x - 2)(3x + 4) &= 15x^2 + 20x - 6x - 8 \\ &= 15x^2 + 14x - 8 \end{aligned}$$

5. $3^{\sqrt{25}} = 3^5 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 = 243$

6. When dividing, the base stays the same and you subtract exponents negative exponents then go to the denominator:

$$\frac{6^2 y^3 z^5}{6^4 y^2 z^7} = \frac{y}{6^2 z^2} = \frac{y}{36z^2}$$

7. When raising a power to a power, you multiply exponents. Any non-zero expression raised to the zero power is 1.

$$\frac{5^0(x^2)^3y}{5x(y^2)^5} = \frac{1x^6y}{5xy^{10}} = \frac{x^5}{5y^9}$$

8. $\frac{7a^3(b+c)^2}{14a(b+c)} = \frac{a^2(b+c)}{2}$

$\frac{(b+c)^2}{b+c}$ works exactly like $\frac{x^2}{x}$. Just as $\frac{x^2}{x} = x$, $\frac{(b+c)^2}{b+c} = (b+c)^1$ or $(b+c)$

9. $3^{2^4} = 3^{16}$

10. $16^{\frac{5}{4}}$ means the 4th root of 16 to the 5th power:

$$(\sqrt[4]{16})^5 = 2^5 = 32$$

11. Remember, to raise a power to a power, multiply exponents.

$$\left(\frac{x^{-\frac{3}{5}}z^{-1}}{2^{-2}z^{-\frac{1}{2}}x} \right)^{-1} = \frac{x^{\frac{3}{5}}z^1}{2^2z^{\frac{1}{2}}x^{-1}}$$

The x^{-1} must go to the numerator and become x^1 :

$$\frac{x^{\frac{3}{5}}x^1z^1}{4z^{\frac{1}{2}}}$$

When multiplying, add exponents. When dividing, subtract exponents.

$$= \frac{x^{\frac{8}{5}}z^{\frac{1}{2}}}{4}$$

12. $7z - 3z + 12 - 5 = 8z + 7$

Combine like terms:

$$4z + 7 = 8z + 7$$

$$\text{Add } -4z \text{ to both sides: } 7 = 4z + 7$$

$$\text{Add } -7 \text{ to both sides: } 0 = 4z$$

$$\text{Divide both sides by 4: } 0 = z$$

13. $-4(3 + 2m) - m = -3m$

Use the distributive law to remove parentheses:

$$-12 - 8m - m = -3m$$

$$-12 - 9m = -3m$$

Add 9m to both sides: $-12 = 6m$

Divide both sides by 6: $-2 = m$

14. $Q = 3x + 2y + 4z$

Add $-2y - 4z$ to both sides in order to isolate the term containing the unknown, x:

$$Q - 2y - 4z = 3x$$

Divide both sides by 3: $\frac{Q - 2y - 4z}{3} = x$

15. $4^x = 8^{x-3}$

Rewrite 4 and 8 as powers of 2: $(2^2)^x = (2^3)^{x-3}$

When raising a power to a power, you multiply exponents: $2^{2x} = 2^{3x-9}$

Hence, $2x = 3x - 9$

$$-x = -9$$

$$x = 9$$

16. Let x = the amount of money invested at 12%.

$x - 4000$ = the amount of money invested at 14%.

Then, $.12x + .14(x - 4000) = 4120$

Multiply each term by 100 to remove decimal points:

$$12x + 14(x - 4000) = 412,000$$

$$12x + 14x - 56000 = 412,000$$

$$26x = 468,000$$

$$x = 18,000$$

He invested \$18,000 at 12% and \$14,000 at 14%

17. Let x = amount of 30% alcohol solution.
 y = amount of 80% alcohol solution.

Equate amount of solution: $x + y = 100$

Equate amount of alcohol: $.30x + .80y = .50(100)$
or $30x + 80y = 50(100)$

If $x + y = 100$, then $y = 100 - x$

$$\begin{aligned} 30x + 80(100 - x) &= 5000 \\ 30x + 8000 - 80x &= 5000 \\ -50x &= -3000 \\ x &= 60 \end{aligned}$$

She needs 60 liters of the 30% solution and 40 liters of the 80% solution.

18. First person takes 9 hours.
Second person takes 6 hours.
Together they take x hours.

Equate what can be done in 1 hour.

$$\frac{1}{9} + \frac{1}{6} = \frac{1}{x}$$

Multiply both sides by the L. C. D. = $18x$:

$$\begin{aligned} 2x + 3x &= 18 \\ 5x &= 18 \end{aligned}$$

$$x = \frac{18}{5} \text{ or } 3\frac{3}{5}$$

Together it would take $3\frac{3}{5}$ hours.

19. $2x + 3y = 10$
 $-3x + 2y = 11$

Multiply the top equation by 3 and the bottom by 2 so that the x terms will be eliminated.

$$\begin{aligned} 6x + 9y &= 30 \\ -6x + 4y &= 22 \\ \hline 13y &= 52 \\ y &= 4 \end{aligned}$$

<p>Substitute $y = 4$ in the first equation: $2x + 3(4) = 10$ $2x + 12 = 10$ $2x = -2$ $x = -1$</p>
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20. $3x + 4y = 8$
 $6x = 7 - 8y$

In the second equation add $8y$ to both sides to get it in standard form:

$$\begin{aligned} 3x + 4y &= 8 \\ 6x + 8y &= 7 \end{aligned}$$

Multiply the top equation by -2 :

$$\begin{aligned} -6x - 8y &= -16 \\ \underline{6x + 8y} &= \underline{7} \\ 0 &= -9 \end{aligned}$$

That cannot happen. The lines are parallel and the system is inconsistent.

21. Let g = price for one gallon of paint
Let b = price for one brush

$$\begin{aligned} 3g + 2b &= 48 \\ 5g + b &= 66 \end{aligned}$$

In the second equation: $b = 66 - 5g$. Substitute this into the top equation:

$$\begin{aligned} 3g + 2(66 - 5g) &= 48 \\ 3g + 132 - 10g &= 48 \\ -7g + 132 &= 48 \\ -7g &= -84 \\ g &= 12 \end{aligned}$$

$$b = 66 - 5g = 66 - 5(12) = 66 - 60 = 6$$

A gallon of pain costs \$12; one brush costs \$6.

22. $\frac{2q + 1}{3} - \frac{q - 1}{4} = -2$

Multiply both sides by 12 to eliminate the denominators:

$$\begin{aligned} 4(2q + 1) - 3(q - 1) &= -2(12) \\ 8q + 4 - 3q + 3 &= -24 \\ 5q + 7 &= -24 \\ 5q &= -31 \end{aligned}$$

$$q = -\frac{31}{5}$$

23. $5x^2 - 3x = 0$

Factor out the common factor of x: $x(5x - 3) = 0$

The product of two factors is 0, and hence one of the factors is 0:

$$x = 0 \text{ or } 5x - 3 = 0$$

$$5x = 3$$

$$x = \frac{3}{5}$$

24. $10x^2 - 13x = 3$

Add -3 to both sides so that 0 is on one side of the equation:

$$10x^2 - 13x - 3 = 0$$

Factor the left hand side:

$$(5x + 1 = 0) (2x - 3) = 0$$

The product of two factors is 0, and hence one of the factors is 0:

$$5x + 1 = 0 \text{ or } 2x - 3 = 0$$

$$5x = -1 \qquad 2x = 3$$

$$x = -\frac{1}{5} \qquad x = \frac{3}{2}$$

25. $2x^2 + 3x = 10$

Add -10 to both sides so that 0 is on one side of the equation:

$$2x^2 + 3x - 10 = 0$$

The quadratic formula says that in the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In the given equation, then, $a = 2$, $b = 3$, and $c = -10$

$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-10)}}{2(2)} = \frac{-3 \pm \sqrt{9 + 80}}{4} = \frac{-3 \pm \sqrt{89}}{4}$$

26. $(2x + 1)(x - 2) = -3$

First multiply the 2 binomials:

$$2x^2 - 3x - 2 = -3$$

Add 3 to both sides:

$$2x^2 - 3x + 1 = 0$$

Factor: $(2x - 1)(x - 1) = 0$

$$2x - 1 = 0 \text{ or } x - 1 = 0$$

$$2x = 1 \qquad x = 1$$

$$x = \frac{1}{2}$$

27. $\frac{y}{3} - \frac{y}{7} = 1$

Multiply both sides by 21:

$$7y - 3y = 21$$

$$4y = 21$$

$$y = \frac{21}{4}$$

28. $\frac{1}{m + 4} - \frac{3}{2m + 8} = -\frac{1}{2}$

Factor the denominators, wherever possible:

$$\frac{1}{m + 4} - \frac{3}{2(m + 4)} = -\frac{1}{2}$$

Multiply both sides by the L. C. D. of $2(m + 4)$:

$$1(2) - 3 = -1(m + 4)$$

$$2 - 3 = -m - 4$$

$$3 = -m$$

$$-3 = m$$

29. $\frac{-6\sqrt{3}}{\sqrt{2}}$

Multiply numerator and denominator by $\sqrt{2}$:

$$\frac{-6\sqrt{3}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-6\sqrt{6}}{2} = -3\sqrt{6}$$

$$30. \quad (2x + 3)^2 = (2x + 3)(2x + 3) = 4x^2 + 6x + 6x + 9 \\ = 4x^2 + 12x + 9$$

$$31. \quad (2x + 3)^2 - (x + 2)^2$$

From problem #30 $(2x + 3)^2 = 4x^2 + 12x + 9$

Then, $(x + 2)^2 = (x + 2)(x + 2) = x^2 + 2x + 2x + 4 = x^2 + 4x + 4$

$$(2x + 3)^2 - (x + 2)^2 = 4x^2 + 12x + 9 - (x^2 + 4x + 4) \\ = 4x^2 + 12x + 9 - x^2 - 4x - 4 \\ = 3x^2 + 8x + 5$$

$$32. \quad 3x^2y - 4xy^2$$

If $x = 3$ and $y = -2$, then the problem becomes:

$$3(3)^2(-2) - 4(3)(-2)^2 = \\ 3(9)(-2) - 4(3)(4) = \\ -54 - 48 = -102$$

33. The distance between points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The given points are $(-4, -2)$ and $(6, -1)$

$$x_1 = -4; \quad x_2 = 6$$

$$y_1 = -2; \quad y_2 = -1$$

Notice that $x_2 - x_1 = 6 - (-4) = 6 + 4 = 10$

$$y_2 - y_1 = -1 - (-2) = -1 + 2 = 1$$

Therefore, $d = \sqrt{10^2 + 1^2} = \sqrt{100 + 1} = \sqrt{101}$

$$34. \quad \frac{8m^4 - 6m^3 + 2m}{2m} = \frac{8m^4}{2m} - \frac{6m^3}{2m} + \frac{2m}{2m} = 4m^3 - 3m^2 + 1$$

$$35. \quad \begin{array}{r} 5p + 7 + \frac{-3}{3p - 2} \\ 3p - 2 \overline{) 15p^2 + 11p - 17} \\ \underline{15p^2 - 10p} \\ 21p - 17 \\ \underline{21p - 14} \\ -3 \end{array}$$

36. Quadrant II

$$37. \frac{6x^2 + 7x - 5}{3x + 5} = \frac{(3x + 5)(2x - 1)}{3x + 5} = 2x - 1$$

$$38. \frac{2p^2 - 5p - 12}{5p^2 - 18p - 8} \cdot \frac{25p^2 - 4}{30p - 12} = \frac{(2p + 3)(p - 4)}{(5p + 2)(p - 4)} \cdot \frac{(5p - 2)(5p + 2)}{6(5p - 2)} = \frac{2p + 3}{6}$$

$$39. \frac{25p^3q^2}{8p^4q} \div \frac{15pq^2}{16p^5}$$

To divide fraction, invert the divisor and multiply:

$$\frac{25p^3q^2}{8p^4q} \cdot \frac{16p^5}{15pq^2} = \frac{10p^3}{3q}$$

40. To divide fractions, invert the divisor and multiply:

$$(y^2 + 2y) \div \frac{y + 5}{y^2 + 4y - 5} = (y^2 + 2y) \cdot \frac{y^2 + 4y - 5}{y + 5}$$

$$= y(y + 2) \cdot \frac{(y + 5)(y - 1)}{y + 5} = y(y + 2)(y - 1)$$

$$41. \frac{3}{t - 2} - \frac{5}{2 - t}$$

Note that $2 - t = (-1)(t - 2)$. The problem can then be written:

$$\frac{3}{t - 2} - \frac{5}{(-1)(t - 2)}$$

$$\text{The two negatives become positive: } = \frac{3}{t - 2} + \frac{5}{t - 2} = \frac{8}{t - 2}$$

$$42. \frac{2}{y + 1} + \frac{6}{y - 1}$$

$$\text{The L. C. D. is } (y + 1)(y - 1): \frac{2(y - 1)}{(y + 1)(y - 1)} + \frac{6(y + 1)}{(y + 1)(y - 1)}$$

$$= \frac{2y - 2 + 6y + 6}{(y + 1)(y - 1)} = \frac{8y + 4}{(y + 1)(y - 1)} \text{ or } \frac{4(2y + 1)}{(y + 1)(y - 1)}$$

$$43. \quad \frac{3r}{10r^2 - 3rs - s^2} + \frac{2r}{2r^2 + rs - s^2}$$

Factor the denominators:

$$\frac{3r}{(5r + s)(2r - s)} + \frac{2r}{(2r - s)(r + s)}$$

The L. C. D. is $(5r + s)(2r - s)(r + s)$

$$\begin{aligned} & \frac{3r(r + s)}{(5r + s)(2r - s)(r + s)} + \frac{2r(5r + s)}{(2r - s)(r + s)(5r + s)} \\ &= \frac{3r^2 + 3rs + 10r^2 + 2rs}{(5r + s)(2r - s)(r + s)} = \frac{13r^2 + 5rs}{(5r + s)(2r - s)(r + s)} \end{aligned}$$

$$44. \quad \frac{\frac{3}{x} - 5}{6 + \frac{1}{x}}$$

In the numerator, the L. C. D. is x and it becomes:

$$\frac{3 - 5x}{x}$$

Similarly, the denominator is:

$$\frac{6x + 1}{x}$$

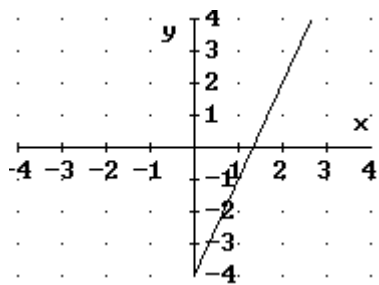
Invert the denominator, and multiply:

$$\frac{3 - 5x}{x} \cdot \frac{x}{6x + 1} = \frac{3 - 5x}{6x + 1}$$

$$45. \quad 3x - y = 4$$

One way to sketch this is to plot 3 points and then connect the points.

- 1) If $x = 0$, then $y = -4$.
- 2) If $x = 1$, then $y = -1$.
- 3) If $x = 2$, then $y = 2$.



46. Slope-intercept form is $y = mx + b$ where m is the slope and b is the y-intercept. This problem states that the slope is $\frac{1}{2}$ and the y-intercept is at $(0, 3)$. Hence, the equation is:

$$y = \frac{1}{2}x + 3$$

47. Slope is defined to be:

$$m = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Given the point $(4, -3)$ and $(-2, -1)$:

$$m = \frac{-3 - (-1)}{4 - (-2)} = \frac{-3 + 1}{4 + 2} = \frac{-2}{6} = \frac{-1}{3}$$

48. The point-slope form of an equation is:

$$m(x - x_1) = y - y_1$$

Given $m = \frac{2}{3}$ and the point $(2, -1)$, the equation is:

$$\frac{2}{3}(x - 2) = y - (-1)$$

$$\frac{2}{3}x - \frac{4}{3} = y + 1$$

Multiply by 3:

$$2x - 4 = 3y + 3$$

$$2x - 3y = 7$$

49. Pts: $(3, 5)$ and $(-1, 0)$ = The slope, $m, = \frac{5 - 0}{3 - (-1)} = \frac{5}{3 + 1} = \frac{5}{4}$

Using point $(-1, 0)$ and the point-slope form from problem 48:

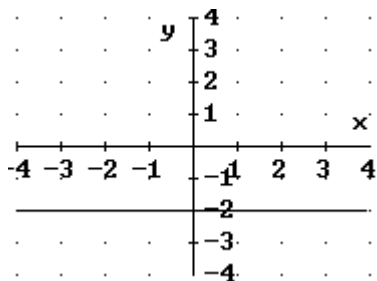
$$\frac{5}{4}(x + 1) = y - 0$$

$$\frac{5}{4}x + \frac{5}{4} = y$$

$$5x + 5 = 4y$$

$$5x - 4y = -5$$

50. Since y is always equal to -2 , the graph is a horizontal line 2 units below the x-axis.



51. Solve $\sqrt{x+2} = x - 4$ Since a square root is involved, square both sides:

$(\sqrt{x+2})^2 = (x-4)^2$ Then you get $x+2 = x^2 - 8x + 16$. Set = 0 getting $x^2 - 9x + 14 = 0$. Factor the quadratic getting $(x-7)(x-2) = 0$. Solving you get $x = 7$ or $x = 2$. 7 checks but 2 does not, so the answer is $x = 7$.

52. Solve $|x-2| < 3$ For absolute value with less than, there are two cases: $x-2 < 3$ and $x-2 > -3$. Solving the first gives $x < 5$; solving the second gives $x > -1$. Therefore $-1 < x < 5$

53. $4! = 4 \times 3 \times 2 \times 1 = 24$

54. $-3x > 12 + x$ Subtract x from both sides gives $-4x > 12$. Divide both sides by -4 giving $x < -3$. Remember, when you multiply or divide an inequality by a negative, the inequality reverses.