REVIEW PROBLEMS FOR COLLEGE ALGEBRA ASSESSMENT TEST-Rev 1

1. Graph the solution of the inequality $x - y < 3$ and $2x + y > -1$

2. Identify the following graphs: a. $x^2 + y^2 = 9$ b. $2x - y = 5$ c. $y = x^2 + 3x - 4$

3. Solve the equation $\frac{4}{2x} - \frac{1}{3x} = 10$

4. Find the solution of the system of equations: $\begin{aligned} 2x + 5y &= 21 \\ 3x - 2y &= -16 \end{aligned}$

5. Expand the determinant: $\begin{vmatrix} 3 & 0 & 1 \\ 2 & 4 & 1 \\ 0 & -2 & 3 \end{vmatrix}$

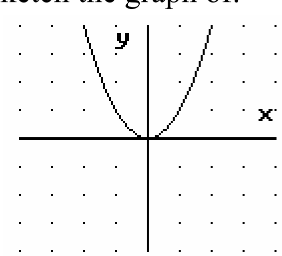
6. Solve the system $\begin{aligned} 3x - 4y &= 1 \\ 2x + 3y &= 12 \end{aligned}$ using Cramer's Rule and determinants.

7. Use the Gauss-Jordan method (matrices) to solve the following system.

$$\begin{aligned} x - y + 5z &= -6 \\ 3x + 3y - z &= 10 \\ x + 3y + 2z &= 5 \end{aligned}$$

8. Describe the left and right hand behavior of $2x^3 - x^2 + 4$.

9. Given the graph $y = x^2$, sketch the graph of:



a. $y = -x^2$
b. $y = x^2 - 3$
c. $y = (x + 2)^2 + 1$

10. Write the general equation of a circle with center (1, -3) and radius 5.

11. What is the vertex of a parabola $y = x^2 - 6x + 11$? Is this value a maximum or a minimum?

12. Find the equation of the line through the point (2, -1) and perpendicular to the line $2x - 3y = 12$. Express the answer in standard form.

13. If $f(x) = 3x - 1$ and $g(x) = x^2 + 3$, then

a. $f(2) =$

b. $g(3) =$

c. $f(g(3)) =$

d. $f(g(x)) =$

e. $g(f(x)) =$

14. Suppose $h(x) = 3x$ and the domain of h is the set of positive integers $\{1,2,3,...\}$. What is the range of h?

15. If $g(x) = \dfrac{x}{x-1}$, what is the domain of g?

16. Use synthetic division to divide $(4x^3 - 9x + 8x^2 - 18)$ by $(x+2)$.

17. Simplify the following radicals:

a. $\sqrt{50}$

b. $\sqrt{32a^2}$

c. $\sqrt{24a^3b^5c^6}$

18. Combine and simplify:

a. $3\sqrt{24} + \sqrt{54}$

b. $3\sqrt{\dfrac{1}{2}} - 2\sqrt{\dfrac{1}{8}}$

c. $9\sqrt{27p^2} - 4\sqrt{108p^2} - 2\sqrt{48p^2}$

19. Rationalize the denominators:

a. $\dfrac{2}{\sqrt[3]{25}}$

b. $\dfrac{4}{3-\sqrt{7}}$

c. $\dfrac{m-4}{\sqrt{m}+2}$

20. Simplify and express with positive rational exponents:

a. $\left(6x^{-2}\right)^{-1}\left(2x^{3}\right)^{-2}$

b. $\left(3p^{-1}q^{-2}\right)^{-1}\left(2pq^{-1}\right)^{-1}$

c. $\dfrac{a^{\frac{1}{4}}}{a^{\frac{3}{4}}b^{\frac{1}{4}}}$

21. For $z \neq 0$, $\dfrac{z^{x}}{z^{x+3}} = ?$

22. Write $c = t^{n}$ in logarithmic form.

23. $\log_{3} 9 =$

24. Write $\log \dfrac{\sqrt{ab}}{c^{2}}$ as a sum, difference, or multiple of logarithms.

25. Combine this expression into a single logarithm. 5 log x + 2 log y – 3 log z

26. Solve for x: $2^{x+1} = 20$ (round to the nearest tenth)

27. If a number added to its reciprocal is two, what is the number?

28. A monthly telephone plan costs $45 plus 6 cents per minute. Write an equation giving the cost for t minutes of phone usage.

29. What is the vertex of the parabola $y = 4(x - 1)^{2} + 10$?

30. $36^{-1/2} = ?$

31. Solve:

a. $-3(x+4)+2 \geq 8-x$

b. $-2 \leq 3k-1 \leq 5$

c. $|4r-5| = 11$

d. $|3x-7| < 4$

e. $|5y+2| > 8$

f. $|3-5t| < 10$

32. If m varies directly as the square root of y and m is 4 when y is 25, what is the value of m when y is 49?

33. Suppose r varies directly as the square of m and inversely as s. If r = 12 when m = 6 and s = 4, find r when m = 3 and s = 10.

34. Simplify:

a. $\sqrt{-144}$

b. $\sqrt{-9} \bullet \sqrt{-36}$

c. $(6+2i)+(4+3i)$

d. $(-1-i)-(-3-4i)$

e. $(2i)(5i)$

f. $(7+3i)(-5+i)$

g. $(2+2i)^2$

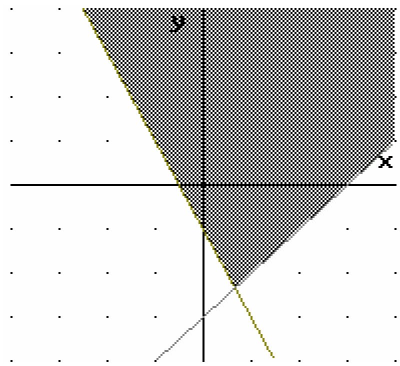
h. $\dfrac{-5}{2-i}$

i. $\dfrac{4-3i}{1+2i}$

35. Ahmed Jones invested some money at 12% and $4000 less than this amount at 14%. Find the amount invested at each rate if his total annual interest income is $4,120.

Solutions to College Algebra Review Problems

1. Graph both inequalities. The solution set is their overlap (intersection) which would be above the two dashed lines.



2. a. circle b. line c. parabola

3. Multiply $\frac{4}{2x} - \frac{1}{3x} = 10$ through by the LCD 6x. After canceling you get 12 - 2 = 60x. Solving this equation gives x = 1/6.

4. 2x + 5y = 21
3x – 2y = -16

One way to solve this system is by elimination. Multiply the top equation by 2 and the bottom equation by 5.

4x + 10y = 42
15x – 10y = -80

Add these two equations together.

19x = -38
x = -2

Substitute x = -2 into the top original equation.

2(-2) + 5y = 21
-4 + 5y = 21
5y = 25
y = 5

The solution is x = -2, y = 5. Written as an ordered pair, this would be (-2,5).

5. $\begin{vmatrix} 3 & 0 & 1 \\ 2 & 4 & 1 \\ 0 & -2 & 3 \end{vmatrix} = 3\begin{vmatrix} 4 & 1 \\ -2 & 3 \end{vmatrix} - 0\begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 4 \\ 0 & -2 \end{vmatrix} = 3(14) - 0(6) + 1(-4) = 38 \quad where \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

Note the sign of the top middle entry switches signs

6. 3x – 4y = 1
 2x + 3y = 12

$$D = \begin{vmatrix} 3 & -4 \\ 2 & 3 \end{vmatrix} = 9 - (-8) = 17$$

$$Dx = \begin{vmatrix} 1 & -4 \\ 12 & 3 \end{vmatrix} = 3 - (-48) = 51$$

$$Dy = \begin{vmatrix} 3 & 1 \\ 2 & 12 \end{vmatrix} = 36 - (2) = 34$$

$$x = \frac{Dx}{D} = \frac{51}{17} = 3 \qquad y = \frac{Dy}{D} = \frac{34}{17} = 2$$

Written as an ordered pair, the answer is (3,2).

7. $x - y + 5z = -6$
 $3x + 3y - z = 10$
 $x + 3y + 2z = 5$

$$\begin{bmatrix} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 1 & 3 & 2 & 5 \end{bmatrix} \xrightarrow{-R_1+R_3} \begin{bmatrix} 1 & -1 & 5 & -6 \\ 3 & 3 & -1 & 10 \\ 0 & 4 & -3 & 11 \end{bmatrix}$$

$$\xrightarrow{-3R_1+R_2} \begin{bmatrix} 1 & -1 & 5 & -6 \\ 0 & 6 & -16 & 28 \\ 0 & 4 & -3 & 11 \end{bmatrix} \xrightarrow{\frac{1}{6}R_2} \begin{bmatrix} 1 & -1 & 5 & -6 \\ 0 & 1 & -8/3 & 14/3 \\ 0 & 4 & -3 & 11 \end{bmatrix}$$

$$\xrightarrow{R_2+R_1} \begin{bmatrix} 1 & 0 & 7/3 & -4/3 \\ 0 & 1 & -8/3 & 14/3 \\ 0 & 4 & -3 & 11 \end{bmatrix} \xrightarrow{-4R_2+R_3} \begin{bmatrix} 1 & 0 & 7/3 & -4/3 \\ 0 & 1 & -8/3 & 14/3 \\ 0 & 0 & 23/3 & -23/3 \end{bmatrix}$$

$$\xrightarrow{\frac{3}{23}R_3} \begin{bmatrix} 1 & 0 & \frac{7}{3} & -\frac{4}{3} \\ 0 & 1 & -\frac{8}{3} & \frac{14}{3} \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{\frac{8}{3}R_3+R_2} \begin{bmatrix} 1 & 0 & \frac{7}{3} & -\frac{4}{3} \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

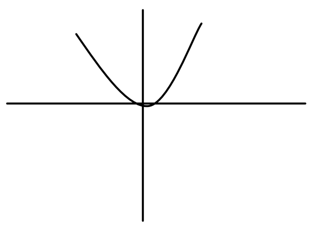
$$\xrightarrow{\frac{-7}{3}R_3+R_1} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \qquad \begin{aligned} x &= 1 \\ y &= 2 \\ z &= -1 \end{aligned}$$

Written as an ordered triplet, this is (1, 2, -1).

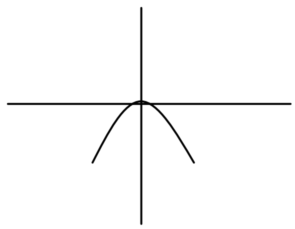
8. $2x^3 - x^2 + 4$

Since the leading coefficient has an odd power (3) the graph will either go up to the right and down to the left or down to the right and up to the left. Since the coefficient of the leading coefficient is positive (2), it will go up to the right and down to the left.

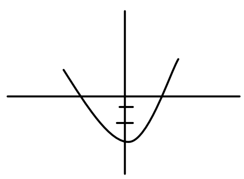
9. y = x^2



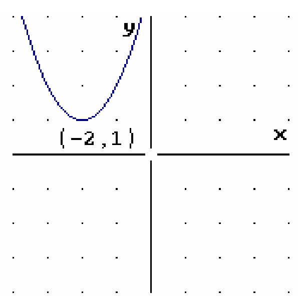
a. $y_2 = -x^2$ will be the reflection of y = x^2 across the x-axis:



b. $y = x^2 - 3$ will take the original graph and move it down 3 units:



c. $y = (x+2)^2 + 1$ will take the original graph and move it to the left 2 units and up 1 unit:



10. Circle with center at (1, -3) and radius 5

$$(x-1)^2 + (y-(-3))^2 = 5^2$$
$$(x-1)^2 + (y+3)^2 = 5^2$$
$$x^2 - 2x + 1 + y^2 + 6y + 9 = 25$$
$$x^2 + y^2 - 2x + 6y - 15 = 0$$

11. $y = x^2 - 6x + 11$

Complete the square: $y = (x^2 - 6x + 9) + 11 - 9 = (x - 3)^2 + 2$

The vertex is at (3, 2) and is a minimum since the graph will open up.

12. $2x - 3y = 12$

Put into slope-intercept form:

$$-3y = -2x + 12$$
$$y = \frac{2x}{3} - 4$$
$$slope = \frac{2}{3}$$

Hence, the slope of a line perpendicular is $-\frac{3}{2}$. The equation of a line with a slope of $-\frac{3}{2}$ and through the point (2, -1) will be:

$$m(x - x_1) = (y - y_1)$$
$$\frac{-3}{2}(x - 2) = (y - (-1))$$
$$\frac{-3}{2}(x - 2) = y + 1$$
$$\frac{-3x}{2} + 3 = y + 1$$
$$-3x + 6 = 2y + 2$$
$$3x + 2y = 4$$

13. $f(x) = 3x - 1,\ g(x) = x^2 + 3$

a. $f(2) = 3(2) - 1 = 5$

b. $g(3) = 3^2 + 3 = 12$

c. $f(g(3)) = f(12) = 3(12) - 1 = 35$

d. $f(g(x)) = f(x^2 + 3) = 3(x^2 + 3) - 1 = 3x^2 + 9 - 1 = 3x^2 + 8$

e. $g(f(x)) = g(3x - 1) = (3x - 1)^2 + 3 = 9x^2 - 6x + 1 + 3 = 9x^2 - 6x + 4$

14. h(x) = 3x Domain of $h = \{1,2,3,...\}$

If the smallest number x can be is 1, then the smallest that h(x) can be is 3 and will continue to be all the positive integral multiples of 3 or {3, 6, 9, 12, . . . }

15. $g(x) = \frac{x}{x-1}$

The domain of g will be all real numbers except x = 1, since 1 is the only number which will cause the denominator to equal 0.

16. $(4x^3 - 9x + 8x^2 - 18) \div (x+2)$

First rewrite in descending powers: $4x^3 + 8x^2 - 9x - 18$

$$\begin{array}{r|rrrr} -2 & 4 & 8 & -9 & -18 \\ & & & & \\ & & -8 & 0 & +18 \\ \hline & 4 & 0 & -9 & 0 \end{array}$$

Answer: $4x^2 - 9$

17. a. $\sqrt{50} = \sqrt{25}\sqrt{2} = 5\sqrt{2}$
b. $\sqrt{32a^2} = \sqrt{16a^2}\sqrt{2} = 4a\sqrt{2}$
c. $\sqrt{24a^3b^5c^6} = \sqrt{4a^2b^4c^6}\sqrt{6ab} = 2ab^2c^3\sqrt{6ab}$

18. a. $3\sqrt{24} + \sqrt{54}$
$= 3(2)\sqrt{6} + 3\sqrt{6}$
$6\sqrt{6} + 3\sqrt{6} = 9\sqrt{6}$

Use: $\sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$
$\sqrt{54} = \sqrt{9}\sqrt{6} = 3\sqrt{6}$

b. $3\sqrt{\frac{1}{2}} - 2\sqrt{\frac{1}{8}}$
$= \frac{3\sqrt{2}}{2} - \frac{2\sqrt{2}}{4}$
$= \frac{3\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Use: $\sqrt{\frac{1}{2}} = \frac{\sqrt{1}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
$\sqrt{\frac{1}{8}} = \frac{\sqrt{1}}{\sqrt{8}} = \frac{1}{2\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{\sqrt{2}}{4}$

c. $9\sqrt{27p^2} - 4\sqrt{108p^2} - 2\sqrt{48p^2}$
$9(3p)\sqrt{3} - 4(6p)\sqrt{3} - 2(4p)\sqrt{3}$
$= 27p\sqrt{3} - 24p\sqrt{3} - 8p\sqrt{3}$
$= -5p\sqrt{3}$

19. a. $\frac{2}{\sqrt[3]{25}} \bullet \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{2\sqrt[3]{5}}{\sqrt[3]{125}} = \frac{2\sqrt[3]{5}}{5}$

b. $\frac{4}{3-\sqrt{7}} \bullet \frac{(3+\sqrt{7})}{(3+\sqrt{7})} = \frac{4(3+\sqrt{7})}{9-\sqrt{49}} = \frac{4(3+\sqrt{7})}{2} = 2(3+\sqrt{7}) = 6+2\sqrt{7}$

c. $\frac{m-4}{\sqrt{m}+2} \bullet \frac{(\sqrt{m}-2)}{(\sqrt{m}-2)} = \frac{(m-4)(\sqrt{m}-2)}{m-4} = \sqrt{m}-2$

20. a. $(6x^{-2})^{-1}(2x^{3})^{-2} = 6^{-1} \bullet x^{2} \bullet 2^{-2} \bullet x^{-6} = \frac{x^{2}}{6 \bullet 2^{2} \bullet x^{6}} = \frac{1}{24x^{4}}$

b. $(3p^{-1}q^{-2})^{-1}(2pq^{-1})^{-1} = 3^{-1}p^{1}q^{2} \bullet 2^{-1}p^{-1}q^{1} = \frac{pq^{2} \bullet q}{3 \bullet 2 \bullet p} = \frac{q^{3}}{6}$

c. $\frac{a^{\frac{1}{4}}}{a^{\frac{3}{4}}b^{\frac{1}{4}}} = \frac{1}{a^{\frac{1}{2}}b^{\frac{1}{4}}}$

21. $\frac{z^{x}}{z^{x+3}} = z^{x-(x-3)} = z^{-3} = \frac{1}{z^{3}}$

22. $c = t^{n}$ so $\log_{t}c = n$

23. $\log_{3} 9 = 2$ since $3^{2} = 9$

24. $\log\frac{\sqrt{ab}}{c^{2}} = \log\frac{\left(a^{\frac{1}{2}}b^{\frac{1}{2}}\right)}{c^{2}} = \log a^{\frac{1}{2}} + \log b^{\frac{1}{2}} - \log c^{2} = \frac{1}{2}\log a + \frac{1}{2}\log b - 2\log c$

25. 5 log x + 2 log y – 3 log z = log x^5 + log y^2 – log z^3 = log $\frac{x^{5}y^{2}}{z^{3}}$

26. Use log of both sides:

$$2^{x+1} = 20$$
$$\log 2^{x+1} = \log 20$$
$$(x+1)\log 2 = \log 20$$
$$x+1 = \frac{\log 20}{\log 2} = \frac{1.3010}{.3010} = 4.322$$
$$x+1 = 4.322$$
$$x = 3.322$$
$$x = 3.3$$

(nearest tenth)

27. A number added to its reciprocal is two gives the equation n + 1/n = 2. Multiply through by the LCD n to get $n^2 + 1 = 2n$. Set = 0 giving $n^2 - 2n + 1 = 0$. Factor, giving $(n-1)^2 = 0$ so n=1.

28. The equation is Cost = 45 + .06t dollars.

29. The parabola $y = 4(x - 1)^2 + 10$ is in the form $y = a(x - h)^2 + k$ where (h,k) is the vertex. Therefore h = 1 and k = 10 so the vertex is (1,10)

30. $36^{-1/2}$ means one over the principal square root of 36 which is 1/6.

31. a. $-3(x+4)+2 \geq 8-x$

$-3x-12+2 \geq 8-x$

$+x \qquad\qquad +x$

$-2x-10 \geq 8$

$+10 \quad +10$

$-2x \geq 18$; ÷ by –2 which will cause the inequality to reverse. (Inequality reverses whenever you multiply or divide by a negative number.)

$x \leq -9$

b. $-2 \leq 3k-1 \leq 5$

Add 1 to each part $-1 \leq 3k \leq 6$

÷ by 3 in each part $-\frac{1}{3} \leq k \leq 2$

c. $|4r - 5| = 11$

4r – 5 = 11 or 4r – 5 = -11

4r = 16   4r = -6

r = 4 or $r = \frac{-6}{4} = \frac{-3}{2}$

d. $|3x-7| < 4$

$-4 < 3x-7 < 4$

$3 < 3x < 11$ (add 7)

$1 < x < 11/3$ (÷ 3)

e. $|5y+2| > 8$

5y + 2 > 8 or 5y + 2 < -8

5y > 6   5y < -10

$y > \frac{6}{5}$ or y < -2

f. $|3-5t| < 10$

-10 < 3 – 5t < 10

-13 < - 5t < 7 (subtract 3)

$\frac{13}{5} > t > -\frac{7}{5}$ or $-\frac{7}{5} < t < \frac{13}{5}$

32. $m = k\sqrt{y}$

$4 = k\sqrt{25}$

$\frac{4}{5} = k$

$m = \frac{4}{5}\sqrt{49}$

$m = \frac{4}{5}(7) = \frac{28}{5}$

33. $r = \frac{km^2}{s}$

$12 = \frac{k(6)^2}{4}$ $\quad 12 = \frac{36k}{4} = 9k$ $\quad \frac{12}{9} = \frac{4}{3} = k$

$r = \frac{\left(\frac{4}{3}\right)(3)^2}{10}$ $\quad r = \frac{(4/3)(9)}{10} = \frac{12}{10} = \frac{6}{5}$

34. Use the fact that $i = \sqrt{-1}$ or $i^2 = -1$

a. $\sqrt{-144} = 12i$

b. $\sqrt{-9} \bullet \sqrt{-36} = (3i)(6i) = 18i^2 = -18$ (This problem must be done in this order.)

c. $(6+2i)+(4+3i) = 6+2i+4+3i = 10+5i$

d. $(-1-i)-(-3-4i) = -1-i+3+4i = 2+3i$

e. $(2i)(5i) = 10i^2 = -10$

f. $(7+3i)(-5+i) = -35+7i-15i+3i^2 = -35-8i-3 = -38-8i$

g. $(2+2i)^2 = (2+2i)(2+2i)$
$= 4+4i+4i+4i^2$
$= 4+8i-4 = 8i$

h. $\frac{-5}{2-i}\left(\frac{2+i}{2+i}\right) = \frac{-5(2+i)}{4-i^2} = \frac{-5(2+i)}{4+1} = \frac{-5(2+i)}{5} = -(2+i) = -2-i$

i. $\frac{4-3i}{1+2i}\left(\frac{1-2i}{1-2i}\right) = \frac{4-8i-3i+6i^2}{1-4i^2} - \frac{4-11i-6}{1+4} = \frac{-2-11i}{5} = -\frac{2}{5} - \frac{11}{5}i$

35. Let x = the amount of money invested at 12%.
x – 4000 = the amount of money invested at 14%.
Then, 0.12x + 0.14(x – 4000) = 4120

Solving this equation, he invested \$18,000 at 12% and \$14,000 at 14%